Pion Decay Constant, Z_A and Chiral Log from Overlap Fermions

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We report our calculation of the pion decay constant f_π, the axial renormalization constant Z_A, and the quenched chiral logarithms from the overlap fermions. The calculation is done on a quenched 20^4 lattice at $a = 0.148$ fm using tree level tadpole improved gauge action. The smallest pion mass we reach is about 280 MeV. The lattice size is about 4 times the Compton wavelength of the lowest mass pion.

1. Numerical Details

For Neuberger's overlap fermion[1], we adopt the following form for the massive fermion action

$$D(m_0) = (\rho + \frac{m_0 a}{2}) + (\rho - \frac{m_0 a}{2})\gamma_5 \epsilon(H) \quad (1)$$

$$\epsilon(H) = H/\sqrt{H^2} \quad (2)$$

$$H = \gamma_5 D_w \quad (3)$$

D_w is the usual Wilson fermion operator, and

$$\rho = -(1/2\kappa - 4) = 1.368 \ \text{ for } \kappa = 0.19 \quad (4)$$

We adopt the optimal rational approximation [2] to approximate the matrix sign function. A deflation algorithm is employed to accelerate the nested loop [2]. We work on a 20^4 lattice with $\beta = 7.60$ tree level tadpole improved Lüscher-Weisz gauge action projecting out 85 smallest eigenmodes of H. We employed shifted matrix inversion method to calculate 16 quark masses ranging from $m_0 a = 0.01505$ to $m_0 a = 0.2736$. As we shall see, the scale determined from f_π is $a = 0.148$ fm which makes the physical length of the lattice to be 2.96 fm. The smallest pion mass turns out to be $\sim 280\,\text{MeV}$ so that the size of the lattice is about 4.1 times of the Compton wavelength of the lowest mass pion.

We adopt the periodic boundary condition for the spatial dimensions and the fixed boundary condition in the time direction so that we can have effectively a longer range of time separation between the source and sink to examine the meson propagators with small quark masses.

2. Zero Mode Effects in Meson Propagators

The quark zero mode is known to contribute to the zero-momentum pseudoscalar propagator such that

$$\begin{aligned}
\int & d^3x \langle \pi(x)\pi(0)\rangle = \\
\int & d^3x [\sum_{i,j=zm} \frac{tr(\psi_j^\dagger(x)\psi_i(x)) tr(\psi_i^\dagger(0)\psi_j(0))}{m_0^2} \\
+ & \ 2 \sum_{i=0,\lambda>0} \frac{tr(\psi_\lambda^\dagger(x)\psi_i(x)) tr(\psi_i^\dagger(0)\psi_\lambda(0))}{m_0(\lambda^2+m_0^2)}] \\
+ & \ \frac{|\langle 0|\pi(0)|\pi\rangle|^2 e^{-m_\pi t}}{2m_\pi} \qquad (5)
\end{aligned}$$

The first term is purely the zero-mode contribution. The second term is the cross term between the zero modes and the non-zero modes. Both zero-mode terms will go down with volume like $1/\sqrt{V}$ and are finite volume artifacts.

The pseudoscalar propagator for a light quark mass ($m_0 a = 0.01915$) in Fig. 1 exhibits a kink at $t/a \sim 8 - 9$.

Zero mode makes the pion propagator fall off faster in t/a at short t. For this reason, we turn

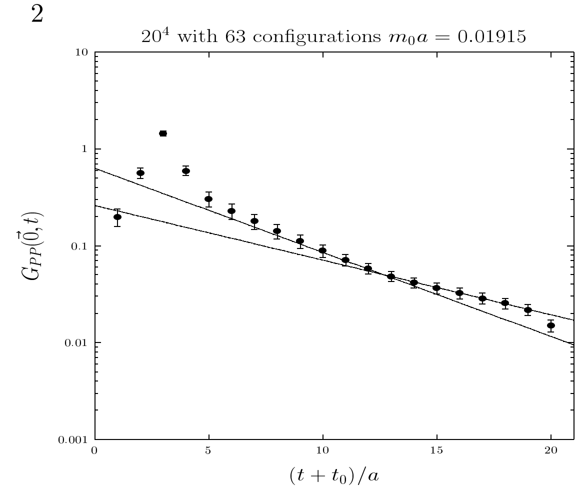

Figure 1. Pion propagator for $m_0a = 0.01915$.

to the propagator $G_{A_4P}(\vec{p}=0,t)$ instead for pion mass. Since the zero modes on one gauge configuration have the same chirality, the pure zero mode contribution vanishes.

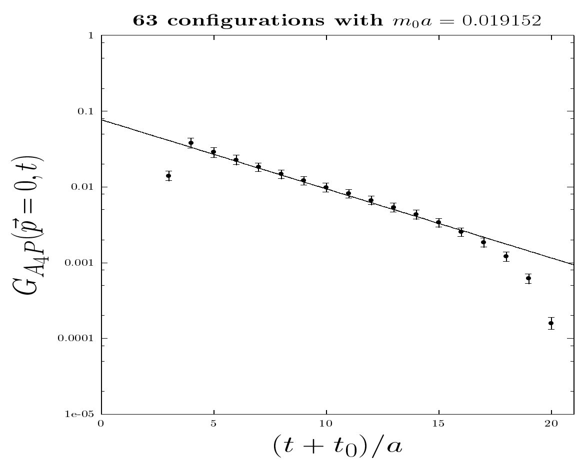

Figure 2. Propagator $G_{A_4P}(\vec{p}=0,t)$ for $m_0a = 0.01915$ for all 63 configurations.

3. Pion Decay Constant f_π and Z_A

We calculate Z_A by the formulas

$$Z_A = \lim_{t\longrightarrow\infty} \frac{2m_0 G_{PP}(\vec{p}=0,t)}{G_{\partial_4 A_4 P}(\vec{p}=0,t)} \qquad (6)$$

$$Z_A = \lim_{t\longrightarrow\infty} \frac{2m_0 G_{PP}(\vec{p}=0,t)}{m_\pi G_{A_4P}(\vec{p}=0,t)}. \qquad (7)$$

The results are shown in the Fig. 3

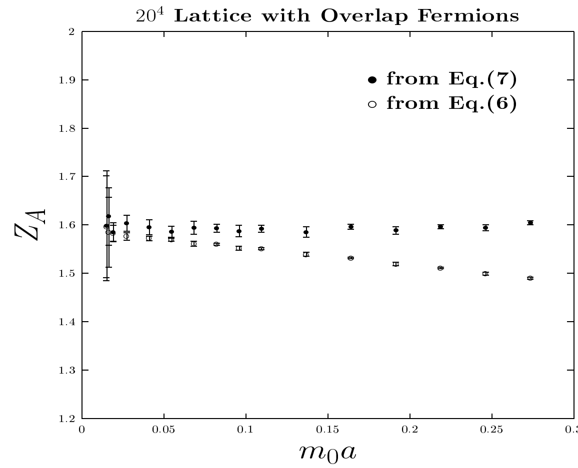

Figure 3. Z_A vs quark mass m_0a with two forms

The difference is because $G_{\partial_4 A_4 P}(\vec{p}=0,t)$ invokes a numerical $O(a^2)$ error. Our final value in chiral limit is $Z_A = 1.589 \pm 0.004$.

We calculate the pion decay constant by using

$$f_\pi a = \lim_{t\longrightarrow\infty} \frac{2m_0a\sqrt{G_{PP}(\vec{p}=0,t)}G_{A_4P}(\vec{p}=0,t)}{\sqrt{m_\pi a}\,G_{\partial_4 A_4 P}(\vec{p}=0,t)} \cdot e^{m_\pi t/2} \qquad (8)$$

$$f_\pi a = \lim_{t\longrightarrow\infty} \frac{2m_0a\sqrt{G_{PP}(\vec{p}=0,t)\,m_\pi a}e^{m_\pi t/2}}{(m_\pi a)^2} \qquad (9)$$

The results are shown in Fig. 4. The difference reflects the numerical $O(a^2)$ in $G_{\partial_4 A_4 P}(\vec{p}=0,t)$. With all 16 data points in a linear fit, we find

$$f_\pi a = 0.0691(11) + 0.109(56)\, m_0 a \qquad (10)$$

Comparing with the experimental value $f_\pi = 92.4\,\mathrm{MeV}$, we determine the scale of our lattice to be $a = 0.148(2)\,\mathrm{fm}$.

4. Quenched Chiral Logs from f_P

In the quenched approximation of QCD, one ignores the virtual quark loops. The study of the anomalous chiral behavior predicted the chiral-log pathologies in the pseudoscalar meson masses[3]. We look for the chiral-log in f_P the

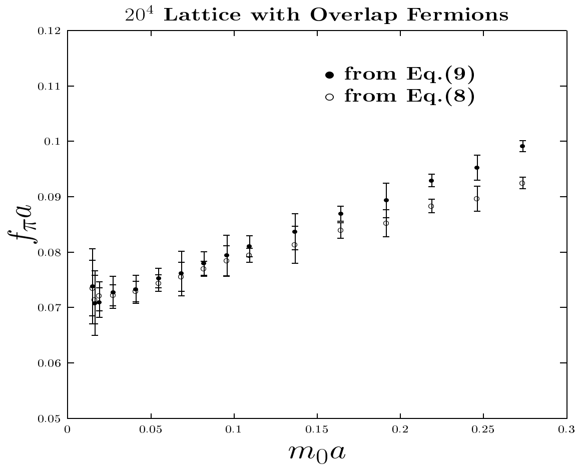

Figure 4. Renormalized $f_\pi a$ vs quark mass $m_0 a$.

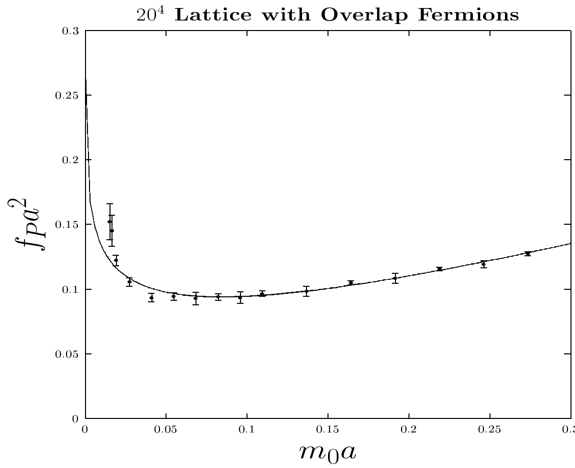

Figure 5. Renormalized $f_P a^2$ vs quark mass $m_0 a$. The solid line is a fit excluding the two smallest quark masses with $\Lambda_\chi = 0.8$ GeV in Eq. (12).

pseudoscalar decay constant.

$$\begin{aligned} f_P &= \langle 0|\bar{\psi} i\gamma_5 \psi|\pi(\vec{p}=0)\rangle \\ &= \lim_{t/a \gg 1} Z_P \sqrt{G_{PP}(t) 2 m_P} e^{m_P t/2} \end{aligned} \tag{11}$$

According to the quenched chiral perturbation theory it should behave as $(\frac{1}{m_\pi^2})^\delta$. For stability we fit the f_P data in the logarithm form.

$$\begin{aligned} & f_P a^2 = \\ & \quad \tilde{f_P} a^2 \{1 - \delta[\ln(A m_0 a/\Lambda_\chi^2 a^2) + 1]\} + B m_0 a \end{aligned} \tag{12}$$

The results are shown in Fig. 5. Excluding the lowest two mass points from fitting results in reasonably small errors and χ^2/DF less than unity are in Table 1.

Table 1
Λ_χ (GeV), $\tilde{f_P} a^2$, δ and χ^2/DF as fitted for $f_P a^2$.

Λ_χ	$\tilde{f_P} a^2$	δ	B	χ^2/DF
0.6	0.083(2)	0.35(4)	0.36(2)	0.93
0.8	0.060(3)	0.48(7)	0.36(2)	0.93

5. Summary

To conclude, we find that the zero mode contribution to the pseudoscalar meson propagators extends to a fairly long distance in the time separation. We obtain the axial renormalization constant, $Z_A = 0.1589(4)$, being fairly independent of $m_0 a$. The renormalized f_π gives $a = 0.148$ fm and the physical lattice size is about 4 times the Compton wavelength of the lowest mass pion. We have observed quenched chiral log in the f_P data set, with δ about 0.4.

This work is partially supported by DOE Grants DE-FG05-84ER40154 and DE-FG02-95ER40907.

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